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THE DYNAMICS OF QUASISTEADY FLOW OF A LIQUID–GAS MIXTURE IN A CONDUIT

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The one-dimensional flow of a liquid–gas mixture is investigated theoretically for the case of a horizontal conduit with phase transition. Approximate solutions have been obtained.

The calculation of the nonsteady flows of gases and liquids in tubes is a complex mathematical problem and is usually accomplished by means of numerical methods [1-4]. Solutions in analytical form have been obtained in [5-7] and the basic quantitative relationships governing one-dimensional nonisothermal quasisteady gas flows have been investigated. Below we will examine the one-dimensional quasisteady nonisothermal flow of a liquid–gas mixture in a horizontal tube of constant cross section. Here we will take into consideration the influence of the phase transition on the process being investigated.

The liquid concentration is characterized by the true φ and flowrate β volumetric concentrations [8]. We will assume that the value of φ at the inlet φ_i to the tube is small and, since the process is nearly steady, the quantity φ is also small:

$$\varphi \ll 1. \tag{1}$$

The quantities φ and β are associated by the equality [8]

$$\varphi = \beta v_m / v_\ell . \tag{2}$$

We will examine only the stratification of the flow which is observed at low liquid concentrations $\beta \lesssim 10^{-2}$ [8]. Since the viscosity of the liquid is considerably greater than the viscosity of the gas and, moreover, the liquid moves near the wall of the tube as the flow becomes separated (in the lower portion), the velocity of the flow v_ℓ must be small in comparison with the gas velocity v. Then

$$\beta \ll \varphi. \tag{3}$$

This assumption is confirmed by experimental results [8]. When $\beta \sim 10^{-2}$ the value of φ is on the order of 10^{-1}. According to these experimental data, however, for the small values of β that we are studying the function $\varphi(\beta)$ is extremely close to the linear. Thus we can assume that

$$\frac{v_\ell}{v_m} = \frac{\beta}{\varphi} = \text{const}. \tag{4}$$

Keeping in mind the smallness of β and φ, instead of (4) we can write

$$\alpha \equiv \frac{v_\ell}{v} = \frac{\beta}{\varphi} = \text{const}. \tag{5}$$

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Let us present the system of equations for the motion of the mixture. In writing the equations of energy, following [9], we will assume that the kinetic energy of the flow changes into the internal energy as a consequence of friction. Thus the friction does not alter the total energy of the mixture. Relying on the results of [8], we can present the continuity and energy equations in the form

$$\frac{\partial}{\partial t}[\varphi\rho_\ell + (1-\varphi)\rho] + \frac{\partial}{\partial X}[\varphi\rho_\ell v_\ell + (1-\varphi)\rho v] = 0, \tag{6}$$

$$\frac{\partial}{\partial t}\left\{\varphi\rho_\ell u_\ell + (1-\varphi)\rho u + \frac{1}{2}[\varphi\rho_\ell v_\ell^2 + (1-\varphi)\rho v^2]\right\} +$$
$$+\frac{\partial}{\partial X}\left\{\varphi\rho_\ell u_\ell v_\ell + (1-\varphi)\rho u v + \frac{1}{2}[\varphi\rho_\ell v_\ell^3 + (1-\varphi)\rho v^3] +\right.$$
$$\left. + p[\varphi v_\ell + (1-\varphi)v]\right\} = \frac{4k}{D}(T^* - T). \tag{7}$$

The existence of a continuous boundary of separation between the gas and the liquid in the case of flow separation enables us to speak of distinct frictional forces for the liquid and the gas at the surface of the tube. For the liquid the expression for the frictional force is written in the form [8]

$$f_\ell = \lambda_\ell \rho_\ell v_\ell^2 (\pi - \theta)(2D\pi\varphi)^{-1}, \tag{8}$$

where θ is the central angle in the cross section of the tube constricting the boundary of phase separation. A similar result is found for the gas

$$f_g = \lambda_g \rho v^2 \theta [2D(1-\varphi)\pi]^{-1}. \tag{9}$$

The angle θ is expressed in terms of the volumetric concentration of the liquid φ. As a consequence of the smallness of φ, the function $\theta(\varphi)$ has the form

$$\theta = \pi - \left(\frac{3}{2}\pi\varphi\right)^{1/3}. \tag{10}$$

The equation for the conservation of momentum, with consideration of expressions (8)-(10), can be written in the form

$$\frac{\partial}{\partial t}[\varphi\rho_\ell v_\ell + (1-\varphi)\rho v] + \frac{\partial}{\partial X}[\varphi\rho_\ell v_\ell^2 + (1-\varphi)\rho v^2] + \frac{\partial p}{\partial X} +$$
$$+\frac{\lambda_g \rho v^2}{2D}[1 - (3\varphi/2\pi^2)^{1/3}] + \lambda_\ell \rho_\ell v_\ell^2 (2D)^{-1}(3\varphi/2\pi^2)^{1/3} = 0. \tag{11}$$

The presence of a phase transition in the flow of the mixture calls for the introduction of the Clapeyron—Clausius equations into our examination:

$$\frac{dp}{dT} = \frac{r}{T}\left(\frac{1}{\rho} - \frac{1}{\rho_\ell}\right)^{-1}. \tag{12}$$

The system of differential equations (6), (7), (11), (12) must be enhanced by the equations of state for the gas

$$p = R\rho T z \tag{13}$$

and for the liquid

$$\rho_\ell = \text{const}, \tag{14}$$

as well as by the empirically established relationship between the velocities of the liquid and gas phases, expressed by equality (5).

We will assume that prior to the initial instant of time t = 0 the flow is steady-state. At the inlet to the tube, when t < 0, the steady-state boundary conditions are given as follows:

$$p(t,\ 0)=p_i,\ T(t,\ 0)=T_i,\ G(t,\ 0)=G_i,\ \varphi(t,\ 0)=\varphi_i. \tag{15}$$

When $t > 0$ at the inlet to the tube and at the outlet from the tube, the quasisteady boundary conditions for pressure are specified:

$$p(t,\ 0)=p_i+\mu p'(t), \tag{16}$$

$$p(t,\ L)=p_0(L)+\mu p''(t). \tag{17}$$

Here μ is a small dimensionless quantity ($\mu \ll 1$); p' and p'' are quantities of the same order of magnitude as p_i.

The presumption of the presence of a phase transition indicates that the temperature at the inlet to the conduit is a single-valued function of pressure. The small changes in temperature must correspond to the small perturbations in pressure and we can therefore write

$$T(t,\ 0)=T_i+\eta(t)\mu T'(t),\ \eta(t)=\begin{cases}0, & t\leqslant 0,\\ 1, & t>0.\end{cases} \tag{18}$$

The temperature of the external medium also undergoes insignificant deviations from the constant value

$$T^*(t,\ X)=T_i^*(X)+\eta(t)\mu T^{*\prime}(X,\ t). \tag{19}$$

When $t > 0$ the constant value (15) of the volumetric concentration of the liquid is retained at the inlet to the conduit.

The condition of steady-state flow under boundary conditions (15) serves as the initial condition.

Let us transform the written system of equations and reduce it to dimensionless form. We note that the internal energy per unit mass of liquid is smaller than the corresponding energy of the gas by the magnitude of the specific heat of vaporization:

$$u_\ell=u-r. \tag{20}$$

We will assume that the absolute temperatures T_i of the mixture and T_i^* of the medium are large in comparison to their difference:

$$\max|T_i^*-T_i|\sim\mu T_i\ll T_i \tag{21}$$

In this case, the change in temperature in the quasisteady flow is insignificant and the heat of vapor formation r can be treated as a linear function of temperature

$$r=r_i-A(T-T_i). \tag{22}$$

Let us introduce the dimensionless variables:

$$(T-T'\mu)/T_i=\vartheta,\ (p-\mu p')/p_i=\psi,\ \rho v/\rho_i v_i=j,\ X/L=y,\ t/t_0=\tau,$$
$$v/v_i=V,\ T^*/T_i=\vartheta^*,\ p'/p_i=\psi',\ p''/p_i=\psi'',\ T'/T_i=\vartheta',$$

where $t_0 = L/v_i$ is the characteristic time of gas motion from the beginning to the end of the conduit.

Equations (6), (7), (11), and (12), with the aid of thermodynamic relationships and equalities (5), (20), and (22), are brought to the form

$$\frac{\partial}{\partial y}\left[\varphi\alpha V\frac{\rho_\ell}{\rho_i}+(1-\varphi)j\right]+\frac{\partial}{\partial\tau}\left[\varphi\frac{\rho_\ell}{\rho_i}+(1-\varphi)\frac{z_i(\psi+\mu\psi')}{z(\vartheta+\mu\vartheta')}\right]=0, \tag{23}$$

$$\left[1-\left(1-\frac{\rho_\ell}{\rho}\right)\varphi\right]\left[\frac{\partial\vartheta}{\partial y}-\frac{Ip_i}{T_i}\frac{\partial\psi}{\partial y}+\frac{v_i^2}{c_pT_i}\left(V\frac{\partial V}{\partial y}+\frac{\partial V}{\partial\tau}\right)+\right.$$
$$\left.+\frac{1}{V}\frac{\partial}{\partial\tau}\left(1-\frac{Rz}{c_p}\right)(\vartheta+\mu\vartheta')-\frac{Ip_i}{T_iV}\frac{\partial}{\partial\tau}(\psi+\mu\psi')\right]-$$

$$-\frac{\rho_\ell}{\rho_i c_p T_i j}\left\{\frac{\partial}{\partial\tau}\varphi[r_i - AT_i(\vartheta+\mu\vartheta'-1)] + \frac{\partial}{\partial y}V\alpha\varphi[r_i -\right.$$

$$\left. - AT_i(\vartheta+\mu\vartheta'-1)] + \frac{v_i^2}{2}(1-\alpha^2)\left(\frac{\partial}{\partial\tau}\varphi V^2 + \alpha\frac{\partial}{\partial y}\varphi V^3\right)\right\} +$$

$$+\frac{Rz_i}{c_p j}\frac{\partial}{\partial y}\{(\psi+\mu\psi')V[1-(1-\alpha)\varphi]\} + \frac{K}{j}(1-\varphi_i)(\vartheta+\mu\vartheta'-\vartheta^*) = 0, \tag{24}$$

$$\frac{v_i^2}{p_i}\left\{\frac{\partial}{\partial\tau}V[\rho_\ell\alpha\varphi+\rho(1-\varphi)] + \frac{\partial}{\partial y}V^2[\rho_\ell\alpha^2\varphi+\rho(1-\varphi)]\right\} +$$

$$+\frac{\partial}{\partial y}(\psi+\mu\psi') + \frac{bz(\vartheta+\mu\vartheta')j^2}{2z_i(\psi+\mu\psi')}\left[1-\left(\frac{3\varphi}{2\pi^2}\right)^{1/3}\right] + \frac{b\lambda_\ell\alpha^2\rho_\ell V^2}{2\lambda_g\rho_i}\left(\frac{3\varphi}{2\pi^2}\right)^{1/3} = 0, \tag{25}$$

$$\frac{d(\psi+\mu\psi')}{d(\vartheta+\mu\vartheta')} = \frac{r_i}{RzT_i}\,\frac{\left[1-\dfrac{AT_i}{r_i}(\vartheta+\mu\vartheta'-1)\right](\psi+\mu\psi')}{\left[\vartheta+\mu\vartheta'-\dfrac{z_i}{z}\dfrac{\rho_i}{\rho_\ell}(\psi+\mu\psi')\right](\vartheta+\mu\vartheta')}, \tag{26}$$

where

$$K = \frac{4kL}{\rho_i v_i D c_p}, \quad b = \frac{\lambda_g v_i^2 L}{Dz_i RT_i}.$$

We will assume that the following conditions are satisfied:

$$\max|1-\vartheta^*| \sim \frac{R}{c_p} \sim \frac{1}{z_i}\left(\frac{\partial z}{\partial\psi}\right)_i \sim \frac{1}{z_i}\left(\frac{\partial z}{\partial\vartheta}\right)_i \sim \frac{v_i^2}{RT_i} \sim \mu \ll 1. \tag{27}$$

As was mentioned in [7], conditions (27) are characteristic for the flow of natural gas through gas pipelines. If an insignificant amount of the liquid phase (condensate) is present in the gas pipeline, additional small quantities appear in the problem, and namely:

$$\varphi_i \sim \alpha \sim \rho_i/\rho_\ell \sim Rz_iT_i/r_i \sim AT_i/r_i \sim \mu \ll 1. \tag{28}$$

Conditions (27) and (28) are also satisfied for subsonic flows of vapor–water mixtures through tubes.

The solution to system (23)-(26) is sought through the method of the small parameter. For functions of the zeroth approximation we have the steady-state boundary conditions. At the inlet to the tube

$$\psi_0(\tau,\ 0) = \vartheta_0(\tau,\ 0) = 1,\ \varphi_0(\tau,\ 0) = 0. \tag{29}$$

At the outlet from the tube

$$\psi_0(\tau,\ 1) = \psi_0(0,\ 1). \tag{30}$$

For correction factors on the order of μ^r the boundary conditions at the inlet to the tube are written in the form

$$\vartheta_r(\tau,\ 0) = \psi_r(\tau,\ 0) = 0, \tag{31}$$

$$\varphi_1(\tau,\ 0) = \varphi_i,\ \varphi_r(\tau,\ 0) = 0\ (r = 2,\ 3,\ \ldots). \tag{32}$$

At the outlet from the tube

$$\psi_1(\tau,\ 1) = \mu(\psi''-\psi') + \psi_1(0,\ 1);\ \psi_r(\tau,\ 1) = \psi_r(0,\ 1)\quad (r = 2,\ 3,\ \ldots). \tag{33}$$

In zeroth approximation of μ, from Eqs. (23)-(26) we obtain

$$\left[1-\varphi_0\left(1-\frac{\rho_\ell}{\rho_0}\right)\right]\frac{\partial\vartheta_0}{\partial y}+\frac{K}{j_0}(\vartheta_0-1)=0, \tag{34}$$

$$\frac{\partial\psi_0}{\partial y}+\frac{b}{2}\frac{j_0^2}{\psi_0}\vartheta_0=0, \tag{35}$$

$$\frac{\partial}{\partial y}\left[\varphi_0V_0\alpha\frac{\rho_\ell}{\rho_i}+(1-\varphi_0)j_0\right]=0, \tag{36}$$

$$\left[1-\frac{AT_i}{r_i}(\vartheta_0-1)\right]\psi_0=0. \tag{37}$$

If the flow in zeroth approximation is isothermal, its characteristics satisfy Eq. (34). In this case, boundary condition (29) for temperature is also satisfied. Equation (37) in the case of isothermal flow is not satisfied. Therefore, we can assume that in zeroth approximation there is no phase transition. In this case Eq. (36) breaks down into two equations: for the gas and the liquid, separately:

$$\frac{\partial}{\partial y}[(1-\varphi_0)j_0]=0, \tag{38}$$

$$\frac{\partial}{\partial y}(\varphi_0V_0)=0. \tag{39}$$

Thus, in zeroth approximation, in the place of system (34)-(37), neglecting the phase transition, we obtain the system (34), (35), (38), (39). This system of equations has a solution which satisfies the above-cited boundary conditions (29) and (30):

$$\vartheta_0=1,\quad \psi_0=(1-by)^{1/2},\quad j_0=1,\quad \varphi_0=0. \tag{40}$$

The equations for the first-order functions are of the form

$$\begin{aligned}&\left(1+\frac{\rho_\ell}{\rho_0}\varphi_1\right)\left[\frac{\partial\vartheta_1}{\partial y}-\frac{I(\psi_0,\vartheta_0)p_i}{T_i}\frac{\partial\psi_0}{\partial y}+\frac{1}{V_0}\frac{\partial}{\partial\tau}(\vartheta_1+\mu\vartheta')\right]+\\&+K(1-\varphi_i)(\vartheta_1+\mu\vartheta'+1-\vartheta^*)+\frac{Rz_i}{c_p}\frac{\partial}{\partial y}(\psi_0V_0)-\\&-\frac{\rho_\ell}{\rho_i c_p T_i}\left[r_i\frac{\partial\varphi_1}{\partial\tau}+\alpha\frac{\partial}{\partial y}(V_0\varphi_1)+v_i^2V_0\frac{\partial V_0}{\partial y}\right]=0,\end{aligned} \tag{41}$$

$$\begin{aligned}&\frac{\partial\psi_1}{\partial y}-\frac{b}{2\psi_0^2}(\psi_1+\mu\psi')+\frac{b}{\psi_0}j_1+\frac{b}{2\psi_0}\left[\vartheta_1+\mu\vartheta'+\right.\\&\left.+\frac{1}{z_i}\left(\frac{\partial z}{\partial\psi}\right)_i(\psi_0-1)-\left(\frac{3\varphi_1}{2\pi^2}\right)^{1/3}\right]+\\&+\frac{b}{2}\frac{\lambda_\ell\alpha^2\rho_\ell}{\lambda_g\rho_i}V_0^2\left(\frac{3\varphi_1}{2\pi^2}\right)^{1/3}+\frac{v_i^2}{RT_iz_i}\frac{\partial V_0}{\partial y}=0,\end{aligned} \tag{42}$$

$$\begin{aligned}&\alpha\frac{\rho_\ell}{\rho_i}\frac{\partial V_0\varphi_1}{\partial y}+\frac{\partial(j_1-\varphi_1)}{\partial y}+\\&+\frac{\partial}{\partial\tau}\left[\varphi_1\left(\frac{\rho_\ell}{\rho_i}-\psi_0\right)+\psi_1+\mu\psi'-\psi_0(\vartheta_1+\mu\vartheta')\right]=0,\end{aligned} \tag{43}$$

$$\frac{Rz_iT_i}{r_i}\frac{d\psi_0}{d(\vartheta_1+\mu\vartheta')}=\psi_0. \tag{44}$$

The relationship between the temperature ϑ_1 and the function ψ_0 follows from Eq. (44) with the boundary condition (31):

$$\vartheta_1 = \frac{Rz_i T_i}{r_i} \ln \psi_0. \tag{45}$$

Equation (44) allows us also to conclude that the temperature increments $\mu\vartheta'$ of second order of smallness correspond to the pressure increments $\mu\psi'$ of first order of smallness. Just as with the other quantities of second order, we will subsequently not take into consideration the quantity $\mu\vartheta'$.

Having substituted expression (45) into Eq. (41), we obtain a first-order equation for the function φ_1:

$$\frac{\partial \varphi_1}{\partial \tau} + \alpha V_0 \frac{\partial \varphi_1}{\partial y} = \left\{ -\alpha \frac{\partial V_0}{\partial y} + \frac{\rho_i c_p T_i}{\rho_0 r_i} \left[\frac{\partial \vartheta_1}{\partial y} - \frac{I(\psi_0, \vartheta_0) p_i}{T_i} \times \right.\right.$$
$$\left.\left. \times \frac{\partial \psi_0}{\partial y} \right] \right\} \varphi_1 + \frac{\rho_i c_p T_i}{\rho_\ell r_i} \left[\frac{\partial \vartheta_1}{\partial y} - \frac{I(\psi_0, \vartheta_0) p_i}{T_i} \frac{\partial \psi_0}{\partial y} + \frac{\mu}{V_0} \frac{\partial \vartheta'}{\partial \tau} + \right.$$
$$\left. + K(\vartheta_1 + 1 - \vartheta^*) \right] - \frac{v_i^2}{r_i \psi_0} \frac{\partial}{\partial y} \frac{1}{\psi_0}. \tag{46}$$

Inequalities (27) allow us to conclude that the quantity Ip_i/T_i which figures in the right-hand side of Eq. (46) is a second-order quantity with respect to μ if $z_i \lesssim 1$. The condition $z_i \lesssim 1$ in actual practice is normally satisfied. It is violated only in the case of exceedingly high pressures.

Having solved Eq. (46) with boundary condition (32) and having eliminated the second-order terms from the solution, we obtain

$$\varphi_1(\tau, y) = \varphi_i \psi_0^{1+\nu} + \frac{1}{\alpha} \psi_0^{1+\nu} \left\{ \frac{\alpha \rho_i}{\rho_\ell} (1 - \psi_0^{-\nu}) - \frac{v_i^2}{r_i (2+\nu)} (1 - \psi_0^{-2-\nu}) + \right.$$
$$+ \frac{\rho_i}{\rho_\ell} \alpha \nu \left[\frac{2}{b(\nu - 2)} \psi_0^{-\nu+2} \left(\ln \psi_0 - \frac{1}{2-\nu} \right) - \frac{2}{b(2-\nu)^2} \right] -$$
$$\left. - \frac{\rho_i c_p T_i}{\rho_\ell r_i} K \left[\int_0^y dy' \vartheta^* \left(\tau - \frac{1}{\alpha} \int_{y'}^{y} \psi_0(x)\, dx,\ y' \right) \psi_0^{-\nu}(y') + \frac{1 - \psi_0^{1-\nu}}{\nu - 1} \right] \right\}, \tag{47}$$

where $\nu = c_p T_i^2 R z_i / \alpha r_i^2$.

Since the functions ϑ_1 and φ_1 are known, Eqs. (42) and (43) should be treated as a system relative to the functions ψ_1 and j_1. In deriving the solution of this system, just as in [7], we will assume satisfaction of the condition

$$b \lesssim \mu^{1/2}. \tag{48}$$

Then, from Eqs. (42) and (43), without consideration of the quantities of the second order of smallness, it follows that

$$\frac{\partial \psi_1}{\partial y} - \frac{b}{2} \left[\psi_1 + \mu\psi' - \vartheta_1 + \left(\frac{3\varphi_1}{2\pi^2} \right)^{1/3} \left(1 - \frac{\lambda_\ell \alpha^2 \rho_\ell}{\lambda_g \rho_i} \right) \right] +$$
$$+ b j_1 + \frac{v_i^2}{R T_i z_i} \frac{\partial}{\partial y} \frac{1}{\psi_0} = 0, \tag{49}$$

$$\frac{\partial j_1}{\partial y} + \frac{\partial \psi_1}{\partial \tau} + \frac{\partial}{\partial \tau} \left[\mu\psi' - \psi_0 \vartheta_1 - \left(\psi_0 - \frac{\rho_\ell}{\rho_i} \right) \varphi_1 \right] -$$
$$- \frac{\partial}{\partial y} \left(\varphi_1 - \frac{\alpha \rho_\ell \varphi_1}{\rho_i \psi_0} \right) = 0. \tag{50}$$

The solution for a system such as (49), (50) under corresponding boundary conditions is presented in [7]. The expression for the function $\psi_1(\tau, y)$ is written in the form

$$\psi_1(\tau, y) = 2\exp\frac{by}{4}\sum_{n=1}^{\infty}\exp\left[-\frac{\tau}{b}\left(\pi^2n^2+\frac{b^2}{16}\right)\right]\times$$

$$\times\left\{\int_0^1 dX\exp\left(-\frac{bX}{4}\right)\psi_1(0, X)\sin\pi nX - \int_0^{\tau} d\tau'\exp\left[\frac{\tau'}{b}\left(\pi^2n^2+\right.\right.\right. \tag{51}$$

$$\left.\left.+\frac{b^2}{16}\right)\right]\left[\int_0^1 dX\exp\left(-\frac{bX}{4}\right)N(\tau', X)\sin\pi nX+\right.$$

$$\left.\left.+\frac{\pi n}{b}(-1)^n\exp(-\delta)(\mu\psi''(\tau')-\mu\psi'(\tau')+\psi_1(0, 1))\right]\right\}\sin\pi ny,$$

where

$$N(\tau, y) = \frac{\partial}{\partial\tau}\left[\mu\psi'-\left(\psi_0-\frac{\rho_\ell}{\rho_i}\right)\varphi_1\right]+\frac{\partial}{\partial y}\left(\alpha\frac{\rho_\ell}{\rho_i}\frac{\varphi_1}{\psi_0}-\frac{\vartheta_1}{2}\right)-$$

$$-\left[1-\frac{1}{6}\left(\frac{3}{2\pi^2}\right)^{1/3}\varphi_1^{-2/3}\left(1-\frac{\lambda_\ell\alpha^2\rho_\ell}{\lambda_g\rho_i}\right)\right]\frac{\partial\varphi_1}{\partial y};$$

$\psi_1(0, y)$ is a first-order correction factor to the pressure ψ in the case of steady flow. This function is a solution of system (49), (50) without the terms containing derivatives with respect to time.

The expression for the function $j_1(\tau, y)$ is obtained from Eq. (49):

$$j_1(\tau, y) = \frac{1}{2}\left[\psi_1+\mu\psi'-\vartheta_1+\left(\frac{3\varphi_1}{2\pi^2}\right)^{1/3}\left(1-\frac{\lambda_\ell\alpha^2\rho_\ell}{\lambda_g\rho_i}\right)\right]-$$
$$-\frac{1}{b}\frac{\partial}{\partial y}\left(\psi_1+\frac{v_i^2}{RT_iz_i\psi_0}\right). \tag{52}$$

The functions ϑ_1, $\psi_1 + \mu\psi'$, j_1 describe the deviations of temperature, pressure, and density of the gas flow from the values of these quantities in the case of steady-state flow in the absence of a liquid phase.

We note that in the flow accompanied by a phase transition the temperature of the mixture in first approximation is a function exclusively of the spatial coordinate, although the process is nonsteady. The exchange of heat with the external medium exerts no influence here.

In the absence of temperature variations for the medium, the distribution of the liquid through the conduit is also steady-state and depends on pressure in zeroth approximation, as well as on the temperature of the medium. The variations in the temperature of the medium result in the appearance of traveling-wave superposition which corresponds to the integral of the function $\vartheta^*\left(\tau-\frac{1}{\alpha}\int_{y'}^{y}\psi_0(X)\,dX\right)$ in expression (47). The velocity u of propagation for these waves is equal to the velocity of the liquid in zeroth approximation

$$u = \alpha v_0(y) \tag{53}$$

and is small in comparison with the velocity of the gas (v_0 is the velocity of the gas in zeroth approximation). The wave amplitude depends on numerous parameters. It increases as the coefficient of heat transfer increases, as well as with an increase in the length of the conduit, and it diminishes as the pressure and density $\rho_i v_i$ of the gas flow increase.

First-order correction factors for pressure are determined from formula (51) which is analogous to the expression for the corresponding quantity in the case of single-phase flow [7]. If the functions ψ', ψ'', $T^{*\prime}$ change smoothly (with a characteristic period $\tau \sim 1$), then, as in the case of single-phase flow, in the expression for ψ_1 we can drop the terms that are small with respect to the parameter b/π, as well as those terms which depend on the initial condition and diminish rapidly with time. As a result, we obtain an extremely simple expression for the function ψ_1 [7]:

$$\psi_1 = y \exp\left[-\frac{b}{4}(1-y)\right][\mu(\psi'' - \psi') + \psi_1(0, 1)]. \quad (54)$$

Thus, the presence of an insignificant amount of liquid, and the phase transition with a smooth relationship between the perturbing factors and time, exert virtually no effect on the pressure distribution.

Expression (52) for the first approximation of gas-flow density contains terms proportional to the root of the third degree from the true volumetric liquid concentration $\varphi_i^{1/3}$, as well as terms that are proportional to pressure fluctuations $\mu\psi'$ and $\mu\psi''$. Consequently, the nonsteady portion of the gas-flow density is a superposition of fluctuations which are caused by pressure perturbations at the inlet to the conduit and at the outlet from the conduit, as well as by the traveling waves that are due to perturbations in the temperature of the medium.

The results obtained here allow us, specifying the concentration of the liquid phase at the inlet to the conduit, to calculate the changes in the flow rate of the liquid and in other flow parameters at the outlet. These problems are of interest in designing and monitoring quasisteady operating regimes in conduits transporting liquid–gas mixtures.

NOTATION

φ, β, the true and flow-rate concentrations of the liquid; G, mass flow rate of the gas; v, v_ℓ, velocities of the gas and of the liquid; v_m, velocity of the mixture; ρ, ρ_ℓ, density of the gas and of the liquid; u, u_ℓ, specific internal energies of the gas and of the liquid; p, pressure; T, T*, temperatures of the mixture and of the medium; k, heat-transfer coefficient; D, inside diameter of the conduit; λ_g, λ_ℓ, coefficients of hydraulic resistance for the gas and for the liquid; r, specific heat of vaporization; X, distance from the beginning of the conduit; t, time; L, length of the conduit; R, gas constant; z, coefficient of compressibility; G_i, v_i, ρ_i, p_i, T_i, T_i^*, z_i, r_i, φ_i, values of the corresponding quantities at the inlet to the conduit; c_p, specific heat capacity of the gas at constant pressure; I, Joule–Thomson coefficient; p_0, pressure at the outlet from the conduit for steady-state flow.

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